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SOLVING GLOBAL TWO DIMENSIONAL ROUTING PROBLEMS
USING SNELL'S LAW AND A* SEARCH

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SOLVING GLOBAL, TWO DIMENSIONAL ROUTING PROBLEMS USING SNELL'S LAW AND A* SEARCH†

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Abstract

Long range route planning based on map data is an important component in the intelligent control system of an autonomous agent. Most attempts to solve this problem rely on applying simple search strategies to high resolution, node and link representations of the map. These techniques have several disadvantages including large time and space requirements. We present a solution technique which utilizes a more intelligent representation of the problem environment. Topographical features are represented as homogeneous cost regions, greatly reducing storage requirements. Given this representation, the A* search strategy is applied to a dynamically created graph that is constructed according to Snell's law. Testing has shown that this strategy reduces time requirements in many cases.

1. Introduction

An important component of any intelligent control scheme for autonomous agents is a route planning ability. The route planning problem has two basic forms. Planning for local motions is the most well studied and most easily solved problem. Local motion planning is generally limited to reasoning about movements within the area local to the autonomous agent, local in the sense that the environment is within scanning range of the agent's sensor equipment. Because of the localized environment, the problem typically assumes a binary nature: every point in the environment is classified as either impassable (part of an obstacle) or passable. To solve this problem, the environment must be described as a set of polygons defining obstacle boundaries. Any point not

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inside an obstacle polygon is traversable. Then, a graph, $G=(V,L)$, is created where V is a set of points including all obstacle vertices as well as the start and goal location [Ref. 1]. The set L contains a link between any two members of V that can be connected by an unobstructed line segment. Given this representation, the problem is reduced to finding the shortest distance path in a graph, a problem that can be solved by several standard techniques. There are several variations on this general approach [Ref. 2,3,4].

The second class of route planning problems is intended to provide routes over long ranges. Here, some form of map is required since the range of movement is too great to be sensed by the on-board equipment of the agent. A more important difference is that the binary nature of the local route planning problem is invalidated. Because of the great diversity that occurs in the physical world, it is not reasonable to assume that all passable areas have the same traversal costs. As an example, riding a bicycle through wet sand requires much more time and effort than riding on a roadway. Thus, the best path for a bicycle rider between two points on a sandy beach could easily be a longer distance roadway route when the alternative is the shortest distance route across the sand.

2. Wavefront Propagation

The best understood and most often used method to solve the long range routing problem is characterized as a *wavefront propagation* technique [Ref 5,6]. To employ this technique, the map provided to the agent must be a lattice. The ratio of the number of points in the lattice to the physical distance represented by the map determines the resolution of the problem. Each lattice point has a link to each of its immediately adjacent neighbors (normally the eight neighboring points, although the degree of connectivity can be any multiple of four). A cost is associated with each link. This cost is meant to represent the cost for the agent to traverse the corresponding area in the physical environment. (Links to points within obstacle areas have infinite cost.) A solution route is generated by first "positioning" the agent at either the start or goal (or both if a bidirectional strategy is employed). Then, the algorithm simulates the passage of time while the agent is allowed to "move" in all directions. The effect is to generate a series of wavefronts, depicting

possible locations for the agent at successive instances of time. When the wavefront reaches the goal, a solution path is retrieved by referencing backpointers, solving wavefront gradients, or a similar technique.

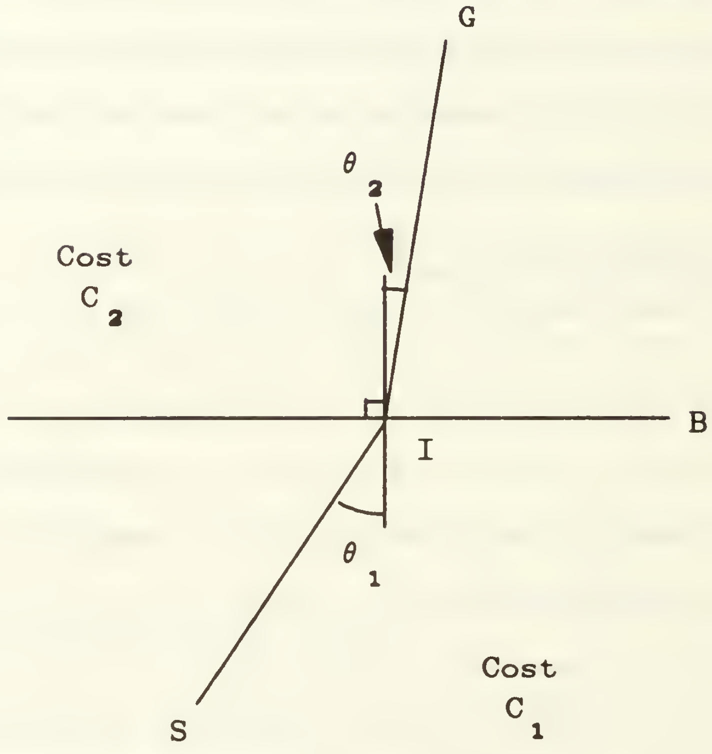
The wavefront technique can solve the long range routing problem. However, there are several drawbacks to using the method [Ref. 7]. First, an omnidirectional search of a high resolution lattice can be expensive computationally. The number of cells that must be examined is roughly proportional to the number of cells in a circle with a radius equal to the number of cells between the start and goal. There is also a *digital bias* inherent in the lattice representation that results in "stair step" approximations to straight line segments [Ref. 6]. Because of this, the method may return a set of digitally equivalent approximations to the optimal path. Another expensive procedure is required to deduce the true, non-digitally biased optimal solution from this set. Finally, most of the search effort is wasted in unproductive portions of the lattice.

3. Snell's Law Based Route Planning

It has been shown that Snell's law, commonly used in optics, also characterizes minimal cost paths in the long range routing problem [Ref. 7,8]. We illustrate the rule in Figure 1. Let B denote a linear boundary so that the cost accrued per unit of distance traveled on one side of B is C_1 and on the other side of B is C_2. Then the (S,I,G) path is the minimal cost path between S and G if and only if $\frac{\sin(\theta_1)}{C_1} = \frac{\sin(\theta_2)}{C_2}$. (Note that θ_1 is the angle between the (S,I) segment and a normal to B while θ_2 is the angle between the (I,G) segment and a normal to B. Also, in Figure 1, C_2 is more expensive than C_1.) This rule is intuitively appealing. Snell's law forces a straight line between S and G to be bent so that increased distance in the low cost region (C_1) is traded for decreased path distance in the high cost (C_2) area.

Application of Snell's law relies on a homogeneous region cost map [Ref. 9] (as opposed to the discrete node and link representation of a continuous environment used in the wavefront technique). Thus, there is no digital bias in the solution paths and the computational cost of the algorithm is not tied to map resolution or the distance between start and goal. However, there are

Figure 1
A Snell's Law Minimum Cost Path



Note: A cost ratio of 2:1 was used to generate this figure

difficulties in applying Snell's law to the homogeneous cost region representation of the environment. First, there is no known closed form solution for a Snell's law problem; using the rule requires iterative search. Secondly, the law allows "blind regions" to exist. Blind regions occur when there are areas on the map that can not be reached by any path obeying Snell's law.

4. A Suitable Problem Representation

The Snell's law based solution method requires a map of the environment where homogeneous cost regions are presented. A homogeneous cost region [Ref. 9] is an area defined by an arbitrary polygon such that all points inside the polygon have identical traversal costs. That is, whenever an agent is inside a homogeneous cost region, the cost for that agent to travel one unit of distance in any direction is constant.

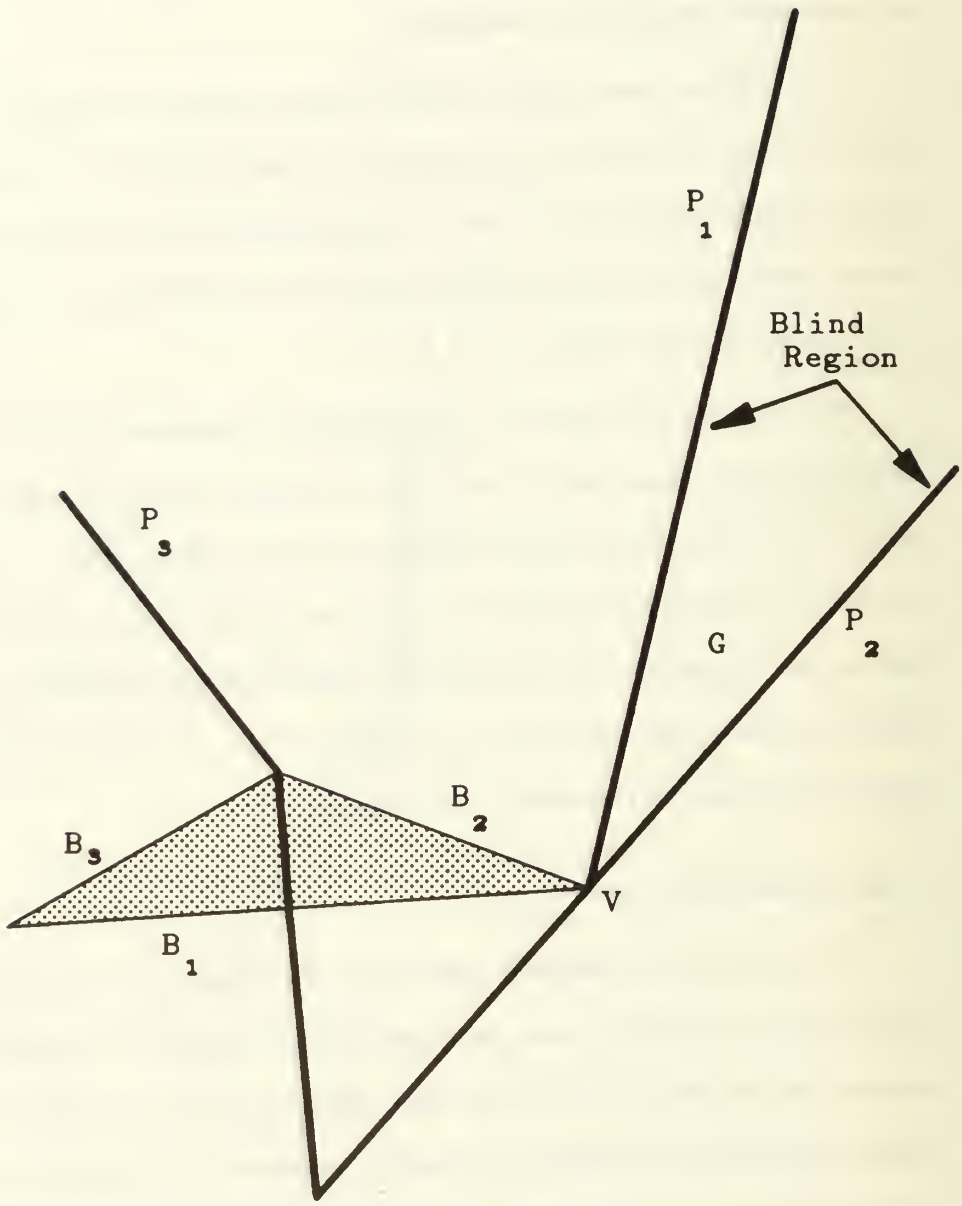
For simplicity of discussion, we assume a ternary classification scheme for constructing the homogeneous cost regions [Ref. 7]. Each point in the physical world is classified into one of three disjoint sets, either obstacle, traversable at high cost, or traversable at low cost. Contiguous areas that have the same traversal classification are grouped together into polygons describing the homogeneous cost regions. The low cost (optimal) regions need not be specifically represented. We assume that these areas constitute the "background" for the map. Thus, obstacle and high cost regions are overlaid onto the optimal cost background.

5. Search and Snell's Law

There is no known closed form solution for a Snell's law problem. Instead, some iterative technique such as bisection or golden section search must be employed. The requirement for search interacts with the fact that Snell's law allows blind regions to exist. Figure 2 exemplifies this dilemma. Suppose that we desire the minimal cost path from S to G and that the high cost region represented by a triangle lies between them. P_1 is a Snell's law path from S through vertex V that is refracted by intersecting sides B_1 and B_2 of the high cost region. P_2 is a path that begins at S and travels infinitesimally close to vertex V so that the high cost region is not intersected. Because of the relative positions of P_1 and P_2, point G is in a blind region. Any Snell's law path that inter-

Figure 2

A Snell's Law Blind Region



Note: A 2:1 cost ratio was used to generate this figure

sects the high cost region to the left of V will be entirely to the left of P_1. An analogous situation exists involving paths to the right of V and P_2. Thus, no Snell's law path from S to G exists and any iterative search based on the law will clearly fail. The only points that can be reached by a Snell's law path involving sides B_1 and B_2 of the high cost region lie in the area between P_3 on the left and P_1 on the right.

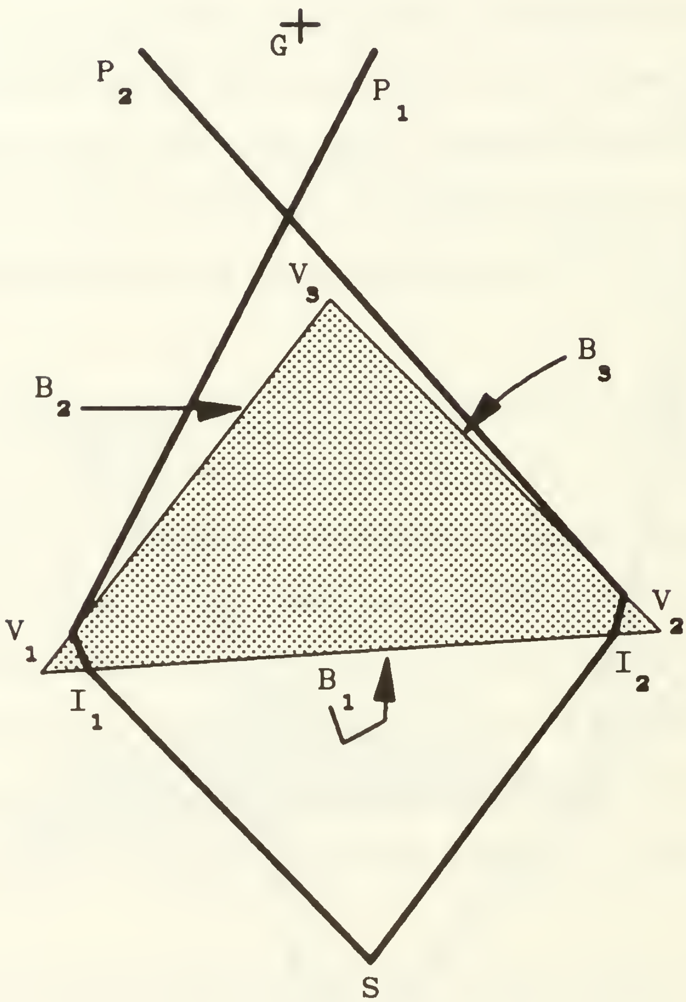
Because of the existence of blind regions, we must insure the success of an iterative search before the search process begins. Thus, we must insure that the search space under consideration "contains" the point that is to be the object of the search. We insure this property by creating "wedges" [Ref. 8] within the search space. Wedges define the portions of the map that can be reached by Snell's law paths from a specified point and involve a specific set of region boundaries. In Figure 2. paths P_3 and P_1 form a wedge that begins at point S and involves sides B_1 and B_2 of the high cost region.

Creating the wedges serves another important purpose. Iterative search techniques use the information gained from one iteration to guide successive attempts. Thus, consecutive attempts must provide consistent information. To guarantee consistency between successive Snell's law paths, we must insure that the paths all intersect exactly the same sequence of region boundaries. In Figure 3, path P_1 intersects boundaries B_1 and B_2 of the high cost region while path P_2 intersects B_1 and B_3. In searching for an S to G path, P_1 indicates that the next attempt should intersect boundary B_1 between V_1 and I_1, while P_2 indicates that the interval between I_2 and V_2 is the most promising. Clearly. the information here is inconsistent and any iterative search technique will be confounded.

To correct the situation in Figure 3, the original wedge formed by P_1 and P_2 should be refined by creating new wedges associated with vertex V_3 of the high cost region. This implies a general principle. Once a wedge has been formed, the information available within that wedge is only guaranteed to be consistent up to the closest unsolved region vertex. A region vertex is unsolved if a Snell's law path (within the wedge) to that vertex has not yet been found. As an example, vertex V_3 in Figure 3 is an unsolved vertex.

Figure 3

Inconsistent Search Information



Notes: A 2:1 cost ratio was used to generate this figure.
Point S is "off center" so that the ray pattern is asymmetric.

Once a Snell's law path to a vertex within a wedge has been found, three new wedges can be formed. In Figure 4, V_3 is the closest unsolved vertex and the Snell's law path from S, across boundary B_1 to V_3 has been found. Any Snell's law path that intersects B_1 to the left of I will intersect B_2 to the left of V_3. Similarly, any Snell's law path intersecting B_1 to the right of I will intersect B_3 to the right of V_3. Thus, to create new wedges guaranteeing the same sequence of region boundaries will be intersected, we split the (S,I,V_3) path into two Snell's law paths at V_3. Figure 5 depicts the result of splitting the path at V_3. Two new paths, P_L (intersecting B_1 and B_2) and P_R (intersecting B_1 and B_3), have been created. They refine the original wedge formed by P_1 and P_2 into new wedges including one formed by P_1 and P_L and another wedge defined by P_R and P_2. There is also a third wedge, that formed by path P_L and P_R. In Figure 5, This last wedge contains no points since P_L and P_R intersect each other immediately at vertex V_3. However, in Figure 2, P_1 and P_2 define a similar wedge that is not empty. Here, the wedge is a blind region and is not directly reachable by any Snell's law path. To reach points inside the blind region, such as G in Figure 2. adherence to Snell's law must be temporarily abandoned at the point initiating the blind region (vertex V in Figure 2). The law is then reapplied in the further search within the wedge (i.e. beyond vertex V in Figure 2).

Thus, we have a general strategy to apply Snell's law in the search for optimal paths. Given an initial wedge, find the closest unsolved vertex that it contains. Find the Snell's law path to the unsolved vertex. Split the path into two paths at the vertex so that the original wedge is refined into three new wedges. Each refinement extends the number of boundaries that are guaranteed to be intersected by any Snell's law path within each new wedge.

6. The Initial Wedge

Given a homogeneous region cost map and a start and goal location, the shortest *distance* start to goal path can be found by ignoring the high cost region polygons and applying binary case methods to the remaining obstacle regions. Clearly, such a path may not have optimal cost, but it must be a feasible solution. The cost of traversing this shortest distance path can be computed by allowing the high cost region polygons back into the cost map. The resulting cost is an upper

Figure 4
A Snell's Law Path to an Unreached Vertex

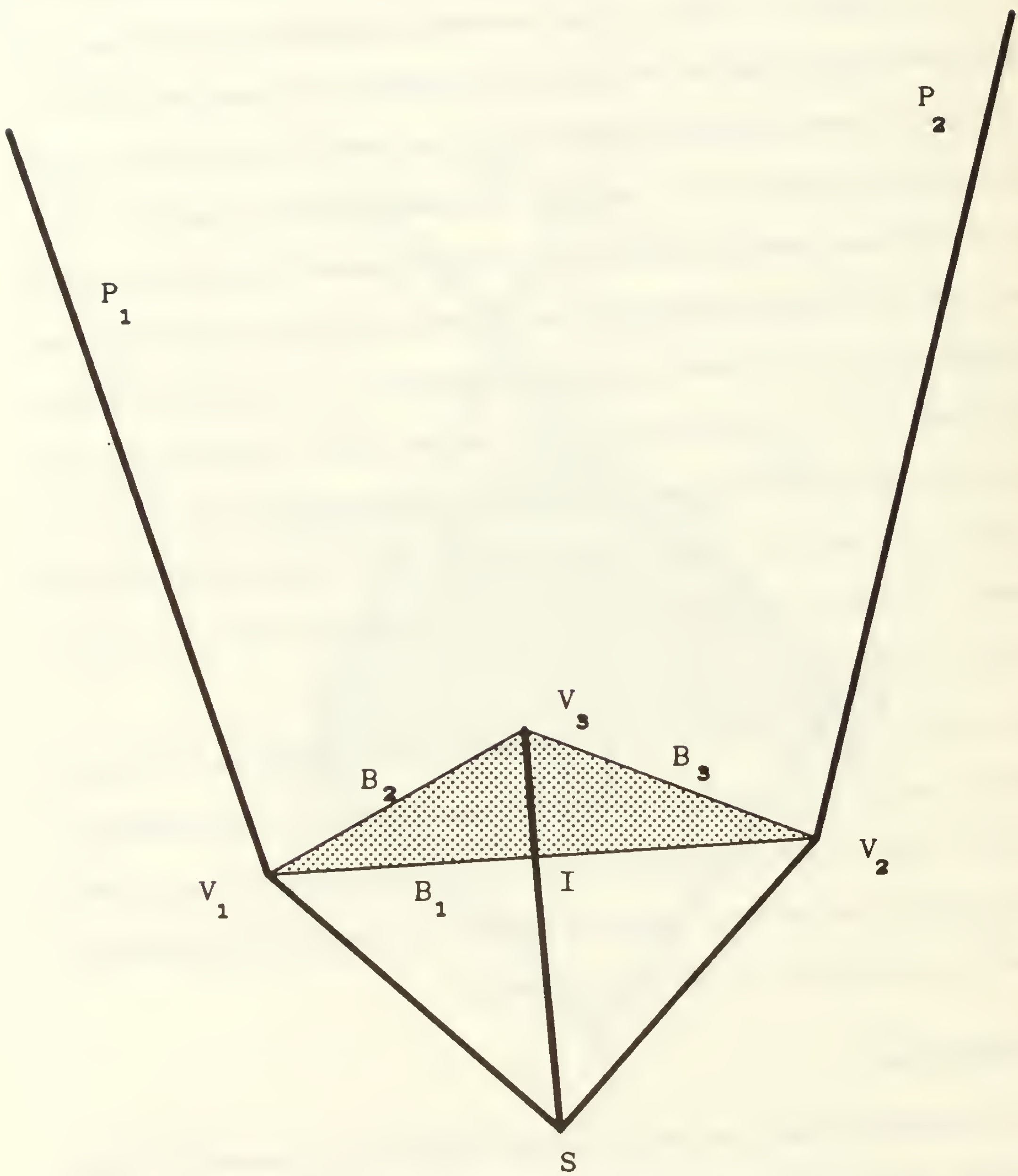


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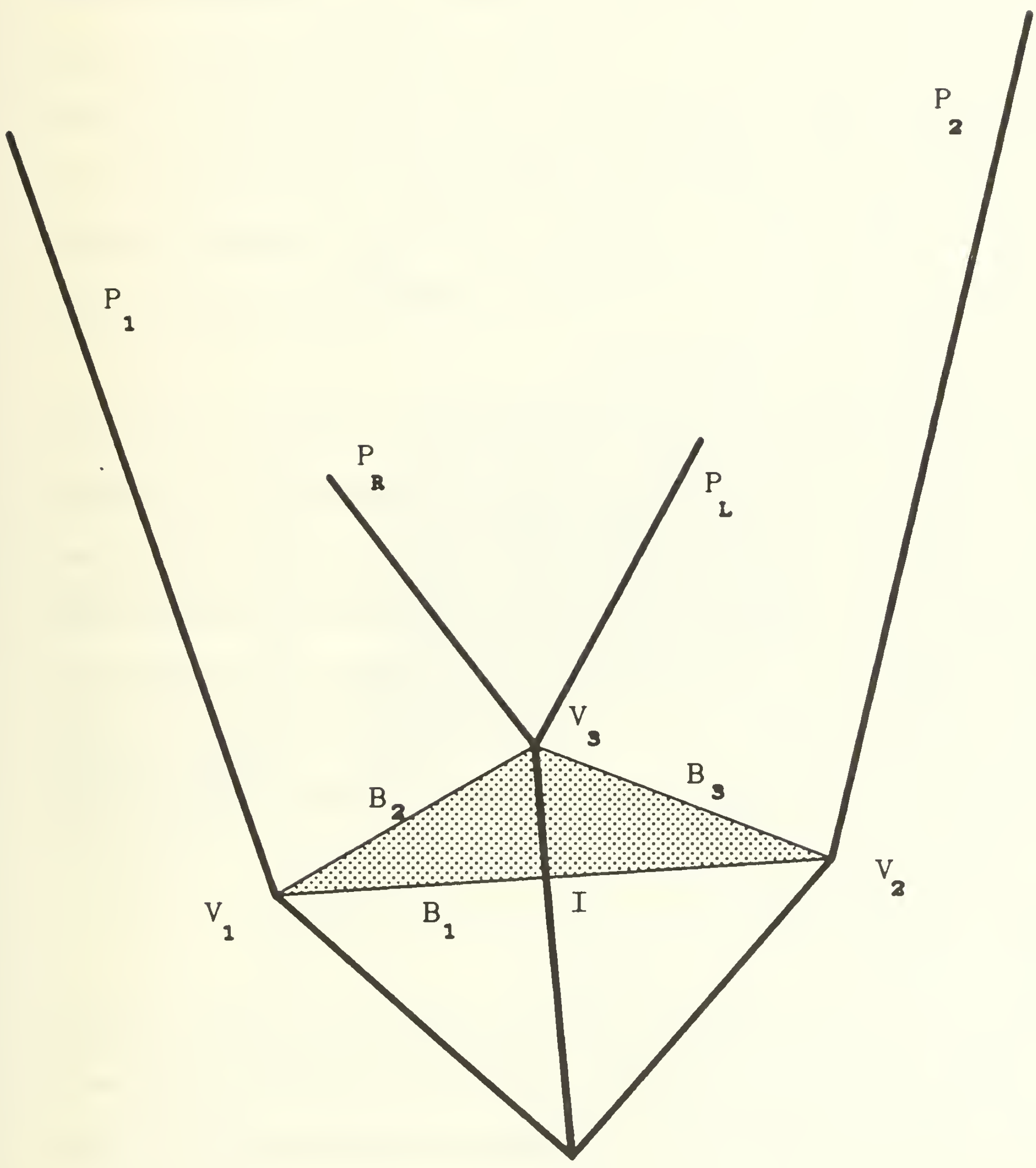
Figure 5

Refinement of the Original Wedge



Note: A 2:1 cost ratio was used to generate this figure.

bound on the cost of the optimal start to goal path. Given an upper bound, physical limits that must contain the optimal path can be constructed. Suppose there is some path from start to goal that stays entirely within the optimal cost regions. The maximum distance that can be traveled along such a path while not accruing cost greater than that of the shortest distance path can be computed. Call this maximum distance the bounding distance. The set of all points such that the distance from the start to that point added to the distance from that point to the goal is equal to the bounding distance defines an ellipse that has the start and goal locations as foci. The optimal path must lie entirely within the physical limits defined by the ellipse boundary. By definition, any path that exits the ellipse must have cost greater than that of the shortest distance path. Note that the ellipse defines the search space for the problem. Only those homogeneous regions that are at least partially within the ellipse need be considered.

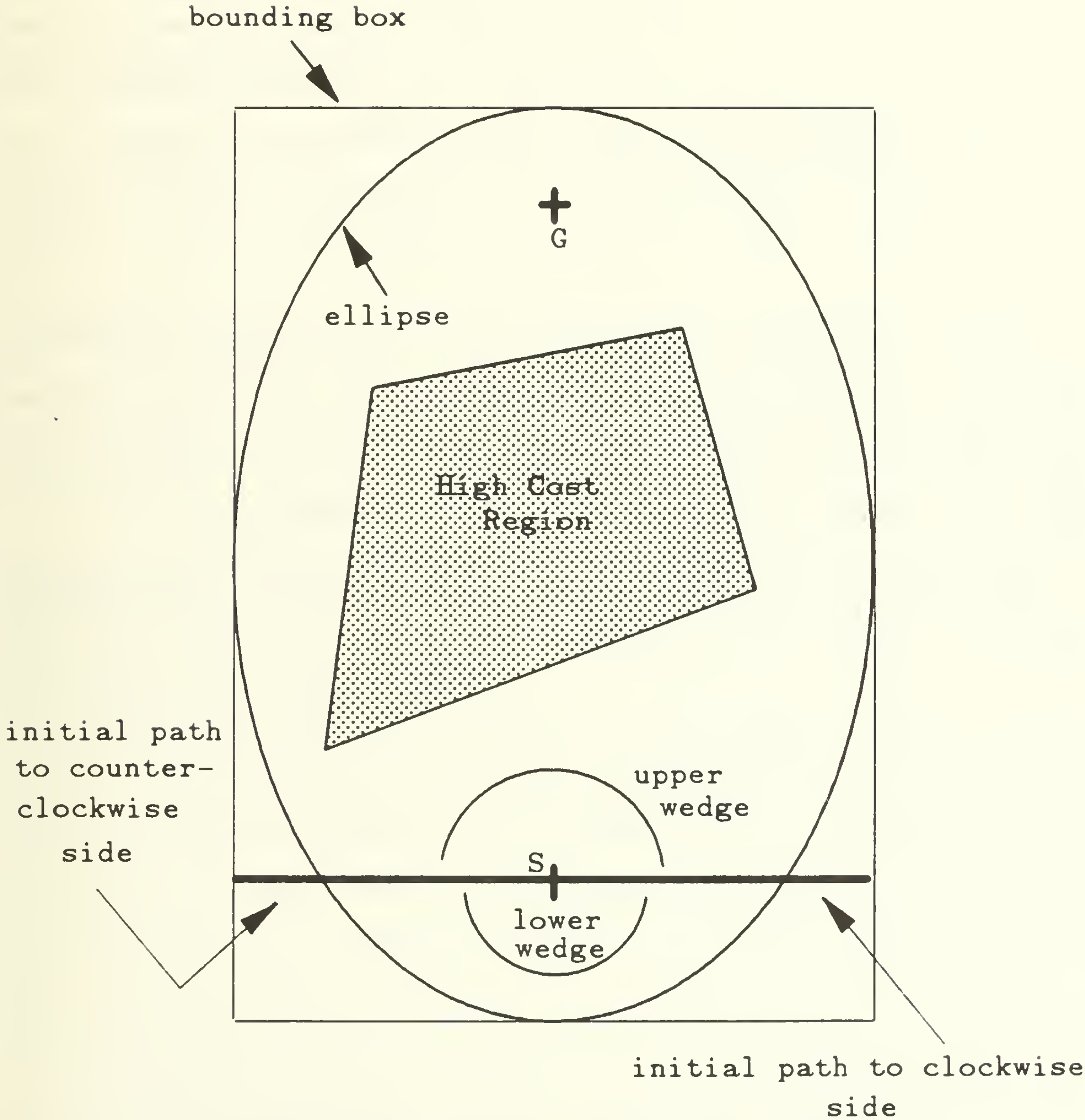
For computational simplicity, circumscribe the ellipse by a rectangle so that each side of the rectangle is tangent to the ellipse. Call this rectangle the bounding box. To create the initial wedges, project one Snell's law path towards a normal intersection with the counter-clockwise side of the bounding box. Project a similar path towards the clockwise side of the bounding box. These two Snell's law paths define two initial wedges, as depicted in Figure 6. It is apparent that the lower wedge of Figure 6 is superfluous. This may not be true in all instances, since the optimal path may occasionally travel away from the goal. As an example, it may be beneficial to take the shortest way out of a high cost area, regardless of the direction to the goal.

7. Pruning Criteria

Given that we can create wedges over a finite space, we can simply continue to refine and search all wedges, finding all feasible (i.e. locally optimal) start to goal paths within the search space. Then, to find the optimal path, select the best feasible solution. However, such a brute force search can be excessive, computationally. Also, it is possible to prune some wedges (and thus their further refinements) from the search space.

First, consider those wedges that are associated with blind regions. Any start to goal path within such a wedge must include the known path from the start to the region vertex at the base

Figure 6
Initialization



Note: The ellipse and bounding box were not generated by actual data. They are for illustrative purposes only.

of the blind region. The true cost of this path can be computed. A lower bound estimate on completing the path is available if we calculate the remaining distance from the region vertex to the goal and assume that this distance could be traveled at the optimal cost. Summing the estimated and the true costs results in a lower bound estimate for any start to goal path within the wedge. If the estimate exceeds the current upper bound cost of the optimal solution, the wedge can be pruned. Also, if there is some other path from the start to the vertex at the base of the blind region, and the other path has a lower start to vertex true cost, the new, higher cost wedge can be pruned since it must be true that the optimal start to goal path is also the optimal path to all points on the path itself. Otherwise, the optimal start to goal path can be "shortcut", resulting in a lower cost path.

Wedges not associated with blind regions can also be pruned. One purpose of building the wedges is that of constantly increasing the known boundaries which must be crossed by Snell's law paths within the wedge. There is a minimum cost path within the wedge that intersects all the known boundaries. The minimum cost path here is readily available without search.

This minimum cost path can be obtained by examining the Snell's law paths defining the wedge as they exit the last known boundary. If both of these paths are rotated in the same direction (either clockwise or counter-clockwise) as they are refracted exiting the last boundary, then the minimum cost path through the wedge is one of these two paths. Thus, comparing the costs of the two paths defining the wedge and selecting the least cost path yields the minimum cost path through the wedge. If the two wedge defining paths are rotated in opposite directions, then a Snell's law path within the wedge that exits the last known boundary at normal (i.e. a zero degree refraction) must exist. This path is the minimum cost path through the wedge and can be easily retrieved. We are able to select the minimum cost paths because the function describing the cost of any path through the wedge is monotonically decreasing as the minimum cost path is approached and monotonically increasing thereafter. Proofs of these properties are beyond the scope of this paper, however, see [Ref. 8] for proofs of very similar results.

Finding the minimum cost path through the wedge provides a lower bound cost for a portion

of a possible start to goal path. We can also compute a lower bound estimate from the last known boundary to the goal by assuming that the minimum distance between them can be travelled at the optimal cost. Summing the two costs provides a lower bound on the total cost of any start to goal path within the wedge. Again, if the lower bound exceeds the current upper bound on the optimal path, the wedge can be pruned.

8. Searching Within Wedges

The pruning criteria presented above also provide methods to rate wedges. The wedge having the lowest estimate of a complete start to goal path should be the first wedge to be refined. Clearly, we have a method well suited to a strategy using a best first agenda and A^* search. The primary difference is that A^* is not used to explore a static, extant graph of nodes and links. Instead, we use the search technique to dynamically create a tree of degree three where each node is a region vertex. The root of the tree is the start location. Leaves of the tree are created when a wedge contains no points (and thus can not be refined further), when a wedge is pruned, or when a start to goal path is located. Each non-leaf node of the tree has one exit arc corresponding to each of the three wedges that can be created based on the Snell's law path to the node (i.e. region vertex) and the wedge of its parent node.

We have now presented the basic concepts required of an algorithm to conduct a Snell's law search over a homogeneous region cost map for the optimal path between two known points. The first step involves finding a feasible solution and its cost. Let this cost be the bounding cost. From the initial solution, create the bounding box and the two initial wedges. Rate the wedges and place them in order of increasing cost estimates on a best first agenda. Exclude from the problem all homogeneous regions that are not at least partially included in the bounding box. Form a set of search points including each remaining region vertex and the goal. Until the agenda is empty or the first wedge on the agenda has a cost estimate exceeding the current upper bound, repeat the following steps.

1) Remove the first wedge from the agenda.

2) Locate the closest unsolved search point within the wedge.

3) Compute the Snell's law path, within the wedge, to the unsolved point.

4) If the unsolved point was the goal, update the bounding cost if appropriate. Otherwise, create three new wedges. Rate each wedge by estimating the lower bound cost of a start to goal path through the wedge. Based on the estimate, either prune the wedge or insert it, in order of increasing estimated cost, into the best first agenda.

9. Implementation

Clearly, there are many details involved in implementing this algorithm that we have not discussed. These include dealing with total internal reflections† and obstacles, methods of finding the closest unsolved point, and similar issues. These difficulties are solvable. However, presenting their solution is beyond the scope of this paper.

A first version of the algorithm has been implemented in C-Prolog (interpretive). The algorithm solves problems given a ternary representation of the homogeneous region cost map. The ternary representation was chosen since it is the simplest scheme that supports the development of a theory to solve n-ary map classifications. (We also note that a ternary representation is appropriate for specific autonomous agents [Ref. 7].) Tests have shown that the algorithm performs well in average cases. Results indicate however, that if the area cost map includes many distinct homogeneous regions within a small area, a wavefront propagation technique is likely to be more appropriate.

Despite the worst case superiority of wavefront techniques, the Snell's law based method has several advantages. First, the method is amenable to parallel execution. The search process within a given wedge is independent from the search of any other wedge. The only communication required is through the agenda. Thus, the algorithm is well suited to computer system architectures that support blackboard strategies. The method is also able to provide feasible solutions

† Total internal reflections can occur when a path exits a high cost region into a lower cost region. The relation expressed by Snell's law may require that the sine of the refraction angle (in the low cost region) be greater than 1. This causes the path to reflect "back onto itself", so that it can not leave the high cost region. Such paths are inappropriate as solutions to the routing problem.

quickly as well as optimal solutions if more time is available. (Testing has shown that as the cost ratio between low and high cost areas increases, an initial solution that treats all cost regions as obstacles closely approximates the cost of the optimal solution.) The method eliminates the problems associated with digitally biased paths and, as a result, returns path descriptions that contain the fewest turn points necessary to accurately describe the optimal path.

Creating and using wedges to segment the map into compartments can also be helpful during execution of a route. If new information requiring changes to the cost map becomes available while the agent is en route, only those wedges interacting with the newly added or deleted homogeneous regions need be recomputed. If the goal changes during execution, only those wedges containing the new goal location need be considered. If the agent strays off course, only the wedges containing the new agent location need be considered.

10. Example Solutions

Figures 7, 8, 9, and 10 depict solutions generated by the Snell's law based algorithm. Each figure features a high cost region (the shaded polygon) overlaid on an optimal cost background. In Figure 7, the high cost region includes both the start and goal locations. The cost ratio of high to low cost is 3:1. Note that the optimal path initially moves away from the goal to quickly exit the high cost region. The path then follows along the region border until a profitable cut-through leading to the goal is found. Figures 8, 9, and 10 all show solutions paths between the same two points involving the same high cost region. In Figure 8, the cost ratio is 2:1 and a portion of the high cost area is included in the optimal path. In Figure 9, the ratio has been changed to 6:1. Note that the optimal solution still contains a portion of the high cost region. although less distance is involved. In Figure 10. a cost ratio of 8:1 was used. At this ratio. the high cost region acts as an obstacle and the shortest distance path around the region constitutes the optimal path.

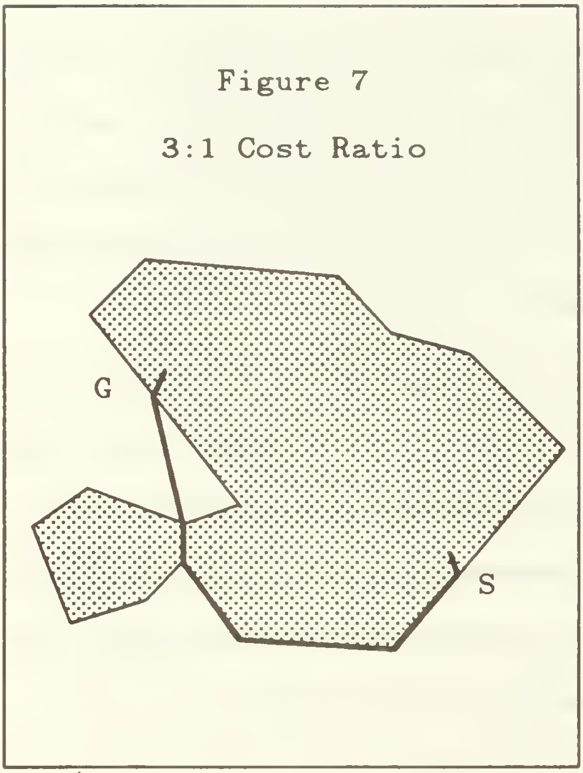


Figure 7

3:1 Cost Ratio

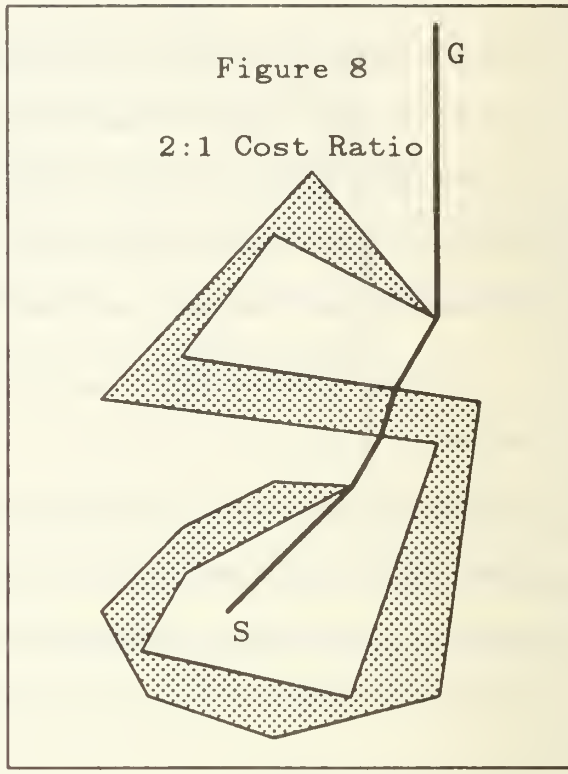


Figure 8

2:1 Cost Ratio

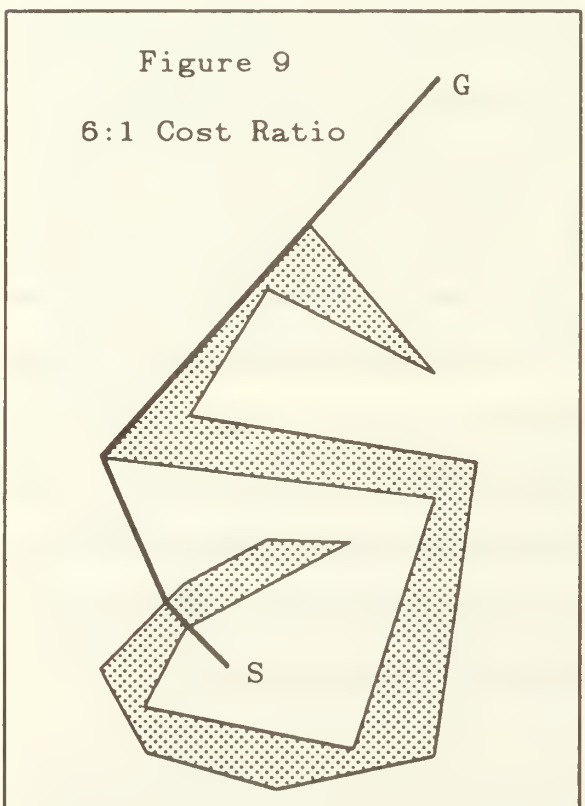


Figure 9

6:1 Cost Ratio

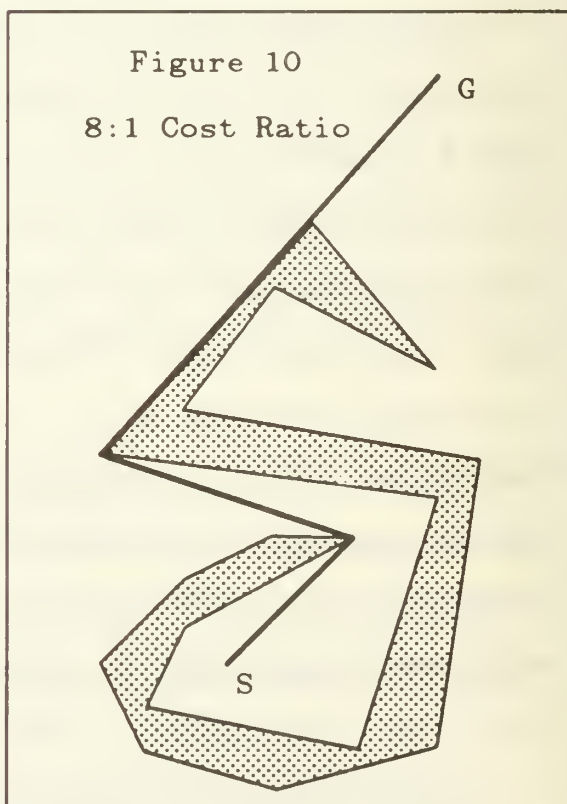


Figure 10

8:1 Cost Ratio

11. Future Work

We have not yet established an order class for the algorithm. Testing indicates that the worst case complexity may be exponential. However, we feel that a worst case would have to be a contrived example. The algorithm performs well in the "average" cases that have been tested to date. An interesting extension is the development of a theory allowing the selection of a particular solution technique (or intermixing techniques), based on the problem at hand. That is, one could allow the system to use wavefront methods when there are many distinct regions in a small area and use the Snell's law method in other cases.

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